Fractal fluctuations in transcranial Doppler signals

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Cerebral blood flow (CBF) velocity measured using transcranial Doppler ultrasonography (TCD) is not strictly constant, but has both a systematic and random component. This behavior may indicate that the axial blood flow in the middle cerebral artery is a chaotic process. Herein we use the relative dispersion, the ratio of the standard deviation to the mean, to show by systematically aggregating the data that the correlation in the beat-to-beat CBF time series is a modulated inverse power law. This scaling of the CBF time series indicates the existence of long-time memory in the underlying control process. We argue herein that the control system has allometric properties that enable it to maintain a relatively constant brain perfusion.
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I. INTRODUCTION

Cerebral autoregulation is the phenomenon of maintaining relatively constant cerebral blood flow (CBF) over a wide range of perfusion pressure and has been well documented in animals and humans [1,2]. However, with the development of technology allowing measurements with high resolution in time, such as transcranial Doppler ultrasonography (TCD) and laser Doppler flowmetry, it has been recognized that regulation of CBF is a dynamical process [3]. Beat-to-beat CBF consists of measurements of a small amount of apparent ''noise'' superimposed on a steady-state mean value. This pattern is similar to other physiological systems, such as beat-to-beat variability of heart rate (HRV). When investigators began processing HRV time series in more detail using nonlinear dynamical techniques [4–8], they discovered that that small amount of ''noise'' had a great deal of information about the cardiac control system. A similar determination was made concerning the fluctuations in the stride interval in a normal human gait. Although the standard deviation in the fluctuations of the gait interval is only approximately 4%, it was found that, like heart rate variability, these fluctuations contain long-term memory and therefore provide information about the underlying control process [9–11]. The time series for both human gait and HRV were determined to be fractal in nature, a consequence of the complex phenomenon that is being controlled. We hypothesize that because of nonlinearity of the complex control system [1,2], regulation of CBF is likely to be a fractal statistical process.

Herein we examine the time series depicting the changes in the cerebral blood flow velocity measured in the middle cerebral artery in normal healthy subjects. Like the ECG and gait time series, the time series of cerebral blood flow velocity consist of a sequence of waveforms. These waveforms are influenced by a complex feedback system involving a number of variables, such as arterial pressure, cerebral vascular resistance [12], plasma viscosity [13], arterial oxygen content [14], arterial CO_2 content [15], as well as other factors [6,16,17].

The variability in the TCD signal was examined by Keunen *et al.* [16] to determine if the apparently random fluctuations in CBF are the result of random influences on the process or if they are the result of chaos. They use the attractor reconstruction technique (ART), along with the Grassberger-Procaccia algorithm (GPA), to analyze continuous waveforms of TCD signals. They found evidence for chaos based on the fact that the correlation dimension, obtained by applying the GPA to the correlation function in the embedding phase space, saturates to a constant value with increasing embedding dimension. They concluded '' . . . the fact that a saturation is observed excludes a random process.'' This conclusion, however, is not necessarily justified. A number of investigators have established that a chaotic time series and colored noise, that is, noise with an inverse power-law spectrum, will be indistinguishable using GPA; see, for example, Osborne and Provenzale [18] and, for a review, West [6]. Both chaos and colored noise processes have fractal dimensions, so that the conclusion reached by Keunan *et al.* [16] is weaker than they believed. This shortcoming was partially corrected in the sequel Keunan *et al.* [17], where the authors made use of the idea of a surrogate data set that could be used to discriminate between chaos and colored noise. The procedure is to randomize the phases between data points, thereby destroying the determinism in a chaotic signal, but not influencing colored noise in any substantial way. In this manner they showed that the phase-space portraits from ART lost their structure in the surrogate TCD data. Therein, they also used the fact that the largest Lyapunov exponent was positive to interpret the TCD time series as chaotic. But again Provenzale *et al.* [19] have established that the K_2 entropy, which is a lower bound on the sum of the Lyapunov exponents, converges to zero for a colored noise process. Therefore, finding a positive largest Lyapunov exponent in a time series, in and of itself, is not sufficient to conclude that the dynamical process is chaotic.

A less ambitious approach to the processing of TCD time-series data was taken by Rossitti and Stephensen [20]. They processed the time series to determine if it is fractal and, rather than analyzing the continuous waveforms of TCD time series, they averaged CBF velocity over 1 s intervals,

and over successive cardiac cycles, to obtain discrete time series. As in other physiological temporal fractals, the loss of complexity in physiological phenomena may be likened to senescence or disease conditions [7]. Rossitti and Stephesen [20] argue that such analysis of TCD data may become useful in clinical diagnosis. Instead of ART and GPA they (Rossitti and Stephesen) used the relative dispersion, the ratio of the standard deviation to the mean, of the time series for a number of levels of aggregation of neighboring data points. If τ_0 is the interval over which the time series is averaged, then the relative dispersion is indicated by $\mathcal{D}(\tau_0)$. If the averaging intervals are now increased to $2\tau_0,3\tau_0,\ldots,n\tau_0$, the relative dispersion in each case is given by $\mathcal{D}(2\tau_0),\mathcal{D}(3\tau_0),\ldots,\mathcal{D}(n\tau_0)$. If the time series is a simple fractal, then the aggregated relative dispersion has a power-law form [7]

$$\mathcal{D}(n\tau_0)=\mathcal{D}(\tau_0)n^{1-D} \tag{1}$$

so that if a plot of the aggregated relative dispersion is made versus the aggregation number n on log-log graph paper, one obtains

$$\ln \mathcal{D}(n\tau_0)=\ln \mathcal{D}(\tau_0)+(1-D)\ln n \tag{2}$$

and the fractal dimension $\mathcal{D}$ is determined by the slope of the straight line. Rossitti and Stephensen [20] obtain fractal dimensions from their discrete time series in the interval $1<\mathcal{D}<1.5$, indicating a fractal random point process (FRPP) with memory.

However, the nonlinear dynamical properties of TCD time series obtained in those previous studies are based on relatively short data segment analysis. Theoretically, the length of the time series should not make a difference in the analysis, because fractal or chaotic time series have no characteristic time scales, so the dynamics of the process can be revealed over any time interval. However, in the real world the fractal character of an experimental time series is only apparent over some longest and shortest time scale, and within this frequency band it is useful to characterize the time series as fractal. We therefore need to distinguish between a mathematical fractal and a physiological fractal [6] for the purposes of data analysis. The TCD time series, especially the continuous waveforms over the time scale of several cardiac cycles, is most likely determined by linear properties of cerebrovascular impedance, rather than by nonlinear regulatory mechanisms [3,21]. Furthermore, analysis based on short data segments by itself, whether fractal or not, may cause unreliable results [7]. Thus, we reason that to reveal the scaling properties of TCD time series, it is necessary to examine the data over an extended period which covers a multitude of different time scales in the TCD time series.

Herein we make use of the relative dispersion of TCD time-series data over a duration of 2 h. For each pulse in the waveform we calculate a mean flow velocity. Thus, τ_0 in our analysis is based on heartbeat numbers. However, we do not restrict our analysis of these data to the assumption of a simple fractal as done to obtain Eq. (1). Instead we determine that the aggregated relative dispersion satisfies a renormalization-group relation whose solution yields, in addition to the inverse power law in Eq. (1), a harmonic modulation of that inverse power law. The presence of this modulation is also evident in the data of Rossitti and Stephensen [20], but they did not mention it in their discussion, focusing instead on the interpretation of the inverse power-law behavior. The existence of this harmonic modulation is strong additional evidence for the scaling or fractal properties of the TCD data as we discuss.

In Sec. II we discuss the acquisition and processing of TCD time series using a renormalization-group approach. In physical processes the solution to scaling equations, as the coarse-grain length scale is changed, is a fixed point of the renormalization-group transform. The repeated application of the renormalization-group operation, which decimates the underlying temporal structure, captures successively the effects of larger and larger scales of fluctuations on the largest-scale variations of interest. An explicit reference to fluctuations of a given scale is eliminated by coarse-graining. Their effects are carried forward implicitly in the parameters of the coarse-grained observable; see, for example Bruce and Wallace [22] for a more complete discussion. In Sec. III the functional form for the relative dispersion developed in Sec. II is fit to the data. The agreement between theory and the TCD time-series data is quite good. In Sec. IV the physiological implications of the results are explored using the properties of universality and scaling.

II. METHODS

A. Subjects

Six healthy subjects (five men and one woman) with a mean age of 29 ± 8 years, height of 177 ± 7 cm, and weight of 76 ± 14 kg, voluntarily participated in the study. All were nonsmokers and were free of known cardiovascular, pulmonary, and cerebrovascular disorders. Each subject was informed of the experimental procedures and signed a written consent form approved by the Institutional Review Boards of The University of Texas Southwestern Medical Center and Presbyterian Hospital of Dallas.

B. Procedures and measurements

Cerebral blood flow velocity in the middle cerebral artery (MCA) was obtained continuously for two hours in the subjects at supine rest, using transcranial Doppler ultrasonography. A typical example of these time series is shown in Fig. 1. This technique allows noninvasive and repeatable estimates of changes in CBF on a beat-to-beat basis. A 2-MHz Doppler probe (DWL Elektronische Systeme) was placed over the temporal window and fixed at a constant angle and position with an adjustable headgear to obtain optimal signals from the MCA according to standard techniques, see American Academy of Neurology [23].

C. Data analysis

Real time beat-to-beat mean values of CBF velocity were calculated as waveform integration of the sampled peak velocity signal within each cardiac cycle divided by the corresponding pulse interval and stored for off-line analysis. We label the mean velocity of the jth beat by X_j, with j

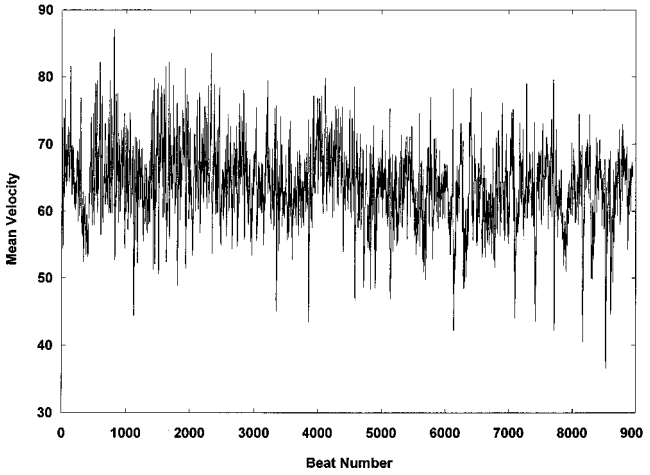

FIG. 1. A typical beat-to-beat CBF velocity time series from one of our subjects is depicted. Note that the time series shown is a 15 min segment of a 2 h data set.

$=1,2,\ldots,N$, and we are interested in the statistics of the beat-to-beat variability of X_j. For N beats the mean is given by

$$\overline{X(1)}=\frac{1}{N}\sum_{j=1}^{N}X_j \tag{3}$$

and the variance is given by

$$\overline{X^2(1)}=\frac{1}{N}\sum_{j=1}^{N}[X_j-\overline{X(1)}]^2 \tag{4}$$

so that the relative dispersion using all the data points is

$$\mathcal{D}_X(1)=\frac{\sqrt{\overline{X^2(1)}}}{\overline{X(1)}}. \tag{5}$$

If the nearest-neighbor intervals (data points) are added together to form a data set of size $N/2$, the relative dispersion is then $\mathcal{D}_X(2)$. Continuing the process of adding the nearest neighbors yields time series with $N/4,\ldots,N/2^n$ data points after n iterations of the procedure, so that we have for the relative dispersions $\mathcal{D}_X(4),\mathcal{D}_X(8),\ldots,\mathcal{D}_X(2^n)$. If the time series is a simple fractal, which is to say it has self-similar statistical properties, we obtain a relation of the form (2) for a log-log graph of the aggregated relative dispersion versus the size of the aggregate.

Equation (1) has the form of the simplest solution to the scaling equation

$$Z(br)=aZ(r), \tag{6}$$

where a and b are parameters to be determined, and $Z(r)$ is an unspecified function; see, for example, West and Deering [24]. Scaling relations of the form (6) have solutions in the same way that differential equations have solutions, which is to say that the dynamics of the underlying phenomenon are determined by finding the general form of the function that obeys the scaling relation and fitting the parameters to the data. One technique for solving such scaling equations is by guessing the form of the solution and determining if the assumed form satisfies the equation. In this spirit we assume the solution

$$Z(r)=A(r)r^{\mu}, \tag{7}$$

which when substituted into Eq. (6) separates into the two equations

$$b^{\mu}=a, \tag{8}$$

$$A(br)=A(r). \tag{9}$$

The first of these relations yields a power-law index $\mu=\ln a/\ln b$. The second of these equations indicates that the function $A(r)$ is periodic in the logarithm of the aggregation size, r, with a period given by $\ln b$. Thus, we conclude that the relative dispersion for a FRPP should have the functional form

$$\mathcal{D}_X(r)=\frac{A(r)}{r^{D-1}}, \tag{10}$$

where we have associated the power-law index μ with the fractal dimension in Eq. (1). Note that the coefficient $A(r)$ in the general solution (10) was assumed to be constant in the fit to the data made by Rossitti and Stephensen. However, a more interesting choice of this function for the beat-to-beat TCD data is

$$A(r)=\exp[\alpha+\lambda\cos(\gamma\ln r)] \tag{11}$$

which is a periodic function in the logarithm of r with period $\ln b$, the latter being related to the parameters in Eq. (11) by $\gamma=2\pi/\ln b$ and $\alpha=\ln a$. The fractal dimension can be expressed in terms of the power-law index by

$$D=1-\mu=1-\ln a/\ln b \tag{12}$$

and where α fixes the overall amplitude in Eq. (10). The equation that is fit to the data on a log-log graph, that replaces Eq. (2), is therefore given by

$$\ln\mathcal{D}_X(m)=\alpha+(1-D)\ln m+\lambda\cos(\gamma\ln m). \tag{13}$$

Here again the fractal dimension is determined by the slope of the fitting curve, but now the curve also has a harmonic modulation in the logarithm of the number of aggregation points in addition to a dominant inverse power law.

III. RESULTS

The renormalization-group model solution (10) is fitted to the data using aggregated relative dispersion on the beat-to-beat variability of the CBF velocity. The parameter values determined from the fitting equation (13) for the modulated inverse power law are listed in Table I. We obtain an average fractal dimension for the six subjects of $D=1.15\pm0.04$, a value not inconsistent with those obtained by the previous investigators. The average fractal dimension and standard deviation is a little lower than that given in Rossitti and Stephesen, 1.24 ± 0.09 for the case of 1 s averages of the time series, but when the heart beat is used as the defining interval for the calculation of mean velocity, such as we use

TABLE I. Renormalization fit to TCD time series data. The best-fit four parameters in the renormalization-group solution (10) to the scaling equation (13) are listed here for the TCD time-series data. Also indicated are the average values of the parameters and their standard deviations over the six subjects.

	α	λ	γ	D
	−3.62	−0.15	1.05	1.19
	−3.66	−0.08	0.90	1.13
	−3.47	−0.18	0.90	1.18
	−3.71	−0.12	1.0	1.14
	−2.88	−0.16	0.90	1.17
	−3.24	−0.05	1.05	1.09
Avg	−3.43	−0.12	0.96	1.15
Std	0.32	0.05	0.08	0.04

here, they obtain 1.17 ± 0.09, with which our results are in complete agreement. In Fig. 2 the relative dispersions given by the data and the fits to these relative dispersions for the six subjects are depicted on log-log graph paper. It is apparent that the dominant behavior is inverse power law, but in each case that behavior is harmonically modulated. The strength of the modulation changes slightly from person to person as does the fractal dimension. However, the curves are more like one another than they are different. This would suggest a normal range of parameter values for healthy individuals. Note that for each subject we have the approximate relation $\ln b = 2\pi(0.96 \pm 0.08)$. Whether this particular value of the period of modulation is significant is too early to tell. The value may be more indicative of the length of the time series than it is of any physiological process. It remains, however, that the period of oscillation indicates the existence of a preferred scale in the coarse-graining procedure, and additional data sets must be tested to determine whether or not this is an artifact.

To determine if the modulated inverse power law is a consequence of noise or of chaos, we implement the surrogate data technique [25] of shuffling the time-series data points to random positions in the sequence for each of the subjects. In Table II the four parameters for the fitting equation (13) are recorded using surrogate data sets. We do this for each of the subjects separately. From this second table we see that the intercepts are virtually unchanged, the λ and γ parameters are zero, and the fractal dimensions all cluster around $D = 1.5$, that is, 1.52 ± 0.03. The nearest-neighbor autocorrelation coefficient [7],

$$r_1 = 2^{3-2D} - 1, \tag{14}$$

allows us to interpret the fractal dimension in terms of the correlation properties of the time series. The fractal dimension $D = 1.5$ implies $r_1 = 0$, so there would be no temporal correlations in surrogate data sets for this fractal dimension. On the other hand, using the average fractal dimension along with its errors, we find $r_1 = 0.58 \pm 0.2$, for the CBF data, indicating a relatively strong correlation between adjacent beat-to-beat variations in the mean CBF velocity. Of course, perfect correlation would be $r_1 = 1$ and would have a fractal dimension of unity according to Eq. (14).

TABLE II. Renormalization fit to surrogate TCD time-series data. The best-fit four parameters in the renormalization-group solution (10) to the scaling equation (13) are listed here for the surrogate data of the time series used to calculate the values in Table I. Each surrogate ensemble consists of 10 members and the averages of the fitting parameters are listed. Also indicated are the average values of the parameters and their standard deviations.

	α	λ	γ	D
	−3.60	0	0	1.54
	−3.69	0	0	1.51
	−3.55	0	0	1.51
	−3.89	0	0	1.48
	−2.80	0	0	1.56
	−3.26	0	0	1.52
Avg	−3.47	0	0	1.52
Std	0.38	0	0	0.03

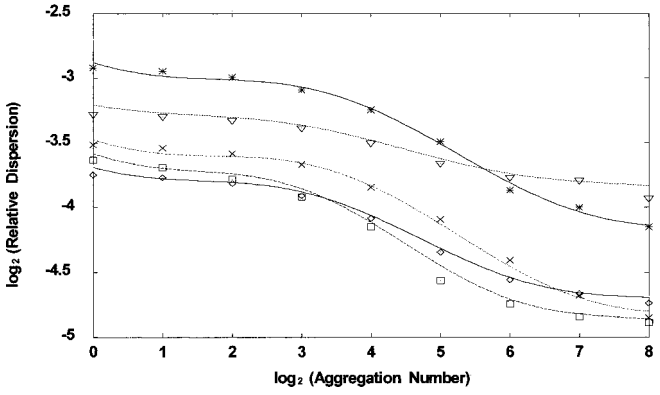

FIG. 2. The logarithm, base 2, of the relative dispersions is plotted versus the exponent of 2 in the numbers of aggregated data points for each of the six subjects. Each time series is 2 h long and consists of between 7×10^3 and 8×10^3 data points. The lines are the best fits to the data and their slopes as well as the other fitting parameters are recorded in Table I. It is clear that the curves define modulated inverse power laws.

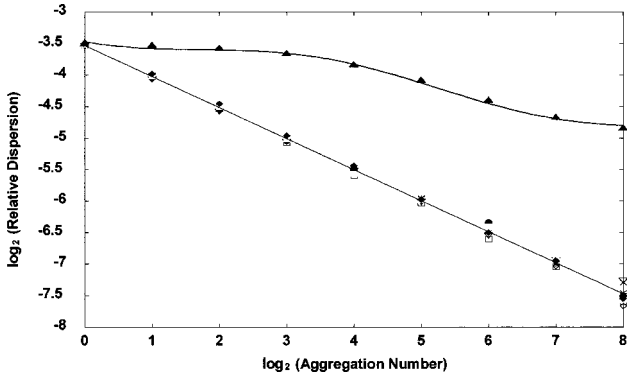

FIG. 3. The logarithm, base 2, of the relative dispersion is plotted versus the exponent of 2 in the number of aggregated data points (●). The solid curve is the best fit to the data and has a slope of -0.15. The dashed curve is the average best fit to ten surrogate data sets and has a slope of -0.52 ± 0.03, with the probability that the difference in the slopes of the two curves can be explained by a linear, additive, uncorrelated random process being $p < 10^{-6}$.

In Fig. 3 the relative dispersion data points for a typical one of the six subjects is shown along with the fit using Eq. (13). That fit is compared with the average fit over an ensemble of ten realizations of the corresponding surrogate time series. The question then arises as to whether the difference between these two fits is statistically significant. We use the fractal dimension as an indicator of the dynamical properties of the TCD time series and determine the level of statistical significance using a *t* test:

$$S=\frac{|D-D_{\rm av}|}{\mathcal{S}}, \tag{15}$$

where $D_{\rm av}$ is the average fractal dimension for the surrogate ensemble and $\mathcal{S}$ is the standard deviation in the fractal dimensions for that ensemble. The probability of observing a significance S or larger if the random process is linear, additive, and uncorrelated is $p=\operatorname{erf} c[S/\sqrt{2}]$ [25]. Thus, in comparing the fractal dimension of each of the experimental time series with that of the average of the corresponding surrogate ensembles, we require a significance level greater than 0.01. This level of significance is achieved with ten realizations in each surrogate ensemble with $S>2.26$. The significance level is determined to be greater than this value in each of the six time series and therefore we observe that the beat-to-beat CBF velocity time series for healthy individuals is a random fractal point process.

It is obvious from Fig. 3 that in addition to the change in slopes (the fractal dimensions between the relative dispersions using the original and the surrogate time-series data) there is also a loss of modulation. This loss of modulation in the relative dispersion is a clear indication that the long-time correlation observed in the original data is a consequence of the underlying dynamics of the phenomenon. This is additional evidence for the fractal nature of the statistical point process.

IV. DISCUSSION

As we pointed out, a fractal random point process (FRPP) is a stochastic process in which the sample paths have a noninteger dimension. To visualize such a process, consider a random walker in a plane, that is, a person who takes a step of a given length at equally spaced time intervals, but whose step direction is uniformly distributed in angle on the interval $(0,2\pi)$. This is the original form of a simple random walk articulated by the biostatistician Pearson [26] and solved by the physicist Lord Rayleigh [27]. The trail the walker leaves behind is quite erratic, so to characterize it we draw concentric circles to enclose the trail, the radius of the circle depending on the time over which the walk has been taking place. In this way the remarkable result that the asymptotic length of the trail is proportional to the radius of the circle raised to a noninteger power is obtained; see, for example, Mandelbrot [28]. The power-law index of the radius yields the fractal dimension of the random walk; see, for example Montroll and West [29]. A simpler way to observe the scaling is by means of the second moment of the dynamical variable X_j at the discrete time j, which for a one-dimensional random walk is

$$\langle X_j^2\rangle \propto j^{2H} \tag{16}$$

and the parameter H is called the Hurst exponent with $0\leqslant H\leqslant 1$. The random-walk approach to processing the data was adopted by Hausdorff *et al.* [9,10] in their analysis of gait and by Peng *et al.* [4] in their analysis of cardiac time series. In the language of random walks, for $1\geqslant H>\frac{1}{2}$. the random walker has a tendency to continue in the direction she is going, so there is persistence to the process. A step in a particular direction is remembered, and the likelihood of the next step being in the same direction is greater than that of reversing directions. This results in a superdiffusive process, one that diffuses more rapidly than normal. In the same way, for $\frac{1}{2}>H\geqslant 0$, the random walker prefers to change her mind with each step, so there is an antipersistence. A step in a particular direction is remembered, and the likelihood of the next step being in the same direction is less than that of reversing directions. This results in a subdiffusive process, one that diffuses more slowly than a normal process. Finally, for $H=\frac{1}{2}$ there is no memory and the random walker is equally likely to step in either direction, no matter what the last step was. This last is a normal diffusion process where the second moment grows linearly in time.

The relation between the Hurst exponent and the fractal dimension of a random time series is well known to be [7]

$$H=2-D \tag{17}$$

so that for the TCD time series we have the average Hurst exponent $H=0.85\pm 0.04$, using the average fractal dimension from Table I. Thus, the beat-to-beat variability in the CBF velocity has a long-time memory and is persistent. The control process for CBF regulation therefore manifests scaling through the long-time correlations of the fluctuations in the CBF velocity. The tying together of the long and short time scales is necessary in order for the feedback to adaptively regulate the complex CBF process to achieve a constant mean velocity in a changing environment.

It may not be clear how the analysis of the continuous waveforms of the TCD signal by Keunan *et al.* [16] reveals the same fractal properties of the underlying phenomenon as the analysis of the discrete TCD time series done herein or that done by Rossitti and Stephenson [20]. To clarify this point, consider the continuous time series $Z(t)$, which we partition into N intervals of equal length τ and average over each of these intervals separately. In this way we obtain the set of discrete values

$$Z_j^{(\tau)}=\frac{1}{\tau}\int_{j\tau}^{(j+1)\tau} Z(t)\,dt, \tag{18}$$

where j is the discrete index that replaces the continuous time and the integral is from the jth to the $j+1$ point in time. Cox [30] showed that if the original time series has an inverse power-law correlation function

$$\mathcal{C}[Z(t)Z(t')]\propto |t-t'|^{-\beta} \tag{19}$$

as established, for example, for the continuous waveforms of the TCD time series by Keunan *et al.*, then the corresponding discrete time series has the correlation function [7]

$$\mathcal{C}[Z_j^{(\tau)}Z_{j+k}^{(\tau)}]=\tfrac{1}{2}[|k+1|^{2-\beta}+|k-1|^{2-\beta}-|k|^{2-\beta}] \quad (20)$$

which asymptotically has the inverse power-law form

$$\mathcal{C}[Z_j^{(\tau)}Z_{j+k}^{(\tau)}]\propto|k|^{-\beta} \quad (21)$$

independently of the averaging interval, τ. Thus, the smoothing done herein in the averaging process does not suppress the scaling present in the original data set. In fact, the process of aggregation emphasizes the scaling behavior of the time series; see, for example, the discussion in West [24].

Regulation of CBF is a complex dynamical process. CBF remains relatively constant over a wide range of perfusion pressure via a variety of feedback control mechanisms, such as metabolic, myogenic, and neurally mediated changes in cerebrovascular impedance responding to the changes in perfusion pressure [1,2]. Furthermore, different regulatory mechanisms may act on different time scales [3]. The fractal characteristics of beat-to-beat variations in the CBF velocity revealed in the present study may indicate that although different regulatory mechanisms may act independently on different time scales, their effects on dynamical changes in CBF may be tied together through scaling. Thus, impairment of one individual component of CBF regulation may influence the overall changes in brain perfusion, but it would not be catastrophic within the compensatory range of CBF regulation.

The interdependence, organization, and concinnity of physiological processes have traditionally been expressed in biology through the principle of allometry. However, this principle, as usually articulated, is static in nature [32], and it is only recently that an attempt to extend the allometry idea to irregular physiological time series in terms of the properties of feedback control have been made [6,31]. An allometric control system achieves its purpose through scaling, enabling a complex system such as the regulation of CBF to be adaptive and accomplish concinnity of the many interacting subsystems. West and collaborators [6,11,24,31] have argued that allometric control is a generalization of the idea of feedback regulation that was implicit in Cannon's concept of homeostasis. The basic notion is to take part of the system's output and feed it back into the input, thus making the system self-regulating by minimizing the difference between the input and the sampled output. More complex systems such as autoregulation of CBF, that involve the elaborate interaction of multiple sensor systems, have more intricate feedback arrangements. In particular, since each sensor responds to its own characteristic set of frequencies, the feedback control must carry signals appropriate to each of the interacting subsystems. The coordination of the individual responses of the separate subsystems is manifest in the scaling of the time series in the output and the separate subsystems select that aspect of the feedback to which they are the most sensitive. In this way an allometric control system not only regulates CBF, but also adapts to changing environmental and physiological conditions.

The idea of the renormalization of a physical process used in the data analysis is that through coarse graining one can determine if the phenomenon under investigation has universality and scaling. By universality we mean that the macroscopic properties of the system are independent of the particular microscopic mechanisms present in the phenomenon. In this way we find that the statistical properties of CBF are the same as those of other complex physiological phenomena [4,6,9–11,31]. Thus, for the purposes here, the particular values of the parameters in Eq. (13), determined from the data, are not significant, except in so far as they indicate that the data scale. Over the long term if we can establish a norm for these parameters, that is, a range of values that can be associated with health and values outside that range can be associated with pathologies, then the values of the scaling parameters for a single individual will be quite important. In fact, these parameters, in particular the fractal dimension, may be used as diagnostics [5].

In summary, we have demonstrated that beat-to-beat fluctuations in CBF velocity are described by a fractal random point process. The allometric properties of TCD time series, as indicated by a modulated inverse power law of the aggregated data, reveal an important property of the cerebral autoregulatory system. Furthermore, this scaling characteristic may be related to different mechanisms of autoregulation and enable a relatively constant brain perfusion under a variety of perturbations of the external environment. Finally, this scaling is consistent with the regulation of CBF being accomplished by means of a low-dimensional, deterministic, nonlinear, dynamical process.

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